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Adaptive control and tracking of chaos in a magnetoelastic ribbon

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We have implemented the tracking of unstable orbits using the full control method of Ott, Grebogi, and Yorke (OGY). (Previous implementations have used only limiting cases of the OGY method.) The implementation is achieved in a mechanical system, the magnetoelastic ribbon. In addition, a method is demonstrated whereby the OGY control parameter may be optimized using only experimental data.

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The control of chaotic systems has generated tremendous interest in the scientific community. Since the pioneering work of Ott, Grebogi, and Yorke [1] demonstrated that chaotic systems could be readily controlled (OGY method), an enormous amount of work has demonstrated that the control of chaos provides a powerful tool for anyone attempting to manipulate chaotic systems (for a review see Refs. [2,3]). Recently adaptive control techniques known as *tracking* have been devised to extend the reach of control of chaos techniques to account for a problem encountered by applied scientists and engineers known as drift or nonstationarity. Schwartz and Triandaf [4] devised an extension to the OGY method which adaptively tracks and maintains control of unstable periodic orbits through large parameter changes. Recent experiments have dramatically demonstrated successful tracking in circuits [5], lasers [6], and, in a remarkable experiment by Petrov and collaborators, in the Belousov-Zhabotinksy (BZ) chemical reaction [7]. The tracking procedure as implemented in electronic circuits by Carroll *et al.* [5] and in multimode lasers by Gills *et al.* [6] is based upon the occasional proportional feedback method (OPF) of chaos control, a limiting case of the OGY method [6]. The implementation of tracking in the BZ reaction [7] is founded on a one-dimensional map-based chaos control algorithm which is a one-dimensional simplification of the OGY algorithm particularly well suited for use in a low-dimensional, highly dissipative chaotic system in combination with Petrov's stability analysis routine.

The main contributions of this paper are the following: (1) experimental demonstration of tracking based on the classic OGY chaos control algorithm; (2) experimental demonstration of tracking in a mechanical system; and (3) an adaptive and very general method of optimizing OGY control that requires very little additional computation and work.

The experimental system consisted of a gravitationally buckled, amorphous magnetoelastic ribbon driven parametrically by a sinusoidally varying magnetic field [8]. The ribbon is clamped at its lower end and its position is measured at a point a short distance above the clamp. The Young's modulus of the ribbon can be varied by more than a factor of 10 by the application of an external magnetic field. We apply an ac magnetic field of amplitude H_{ac} and frequency f added to a dc field of amplitude H_{dc}, such that $H_{applied}(t)=H_{dc}+H_{ac}\sin(2\pi ft)$. In this experiment we choose $f=1.18$ Hz, $H_{ac}=1.05$ Oe and typically H_{dc} is between -1.02 and -1.80 Oe. To implement the OGY control algorithm, we measure the position ξ_j of a point on the ribbon once every driving period. We then construct a delay coordinate embedding by plotting the current position ξ_j vs ξ_{j-d}, where d is the delay.

In the chaotic regime we identify unstable periodic orbits of period d by looking for saddle points lying on the diagonal, $\xi_j=\xi_{j-d}$. In order to control the system onto one of these unstable periodic fixed points $\boldsymbol{\xi}_f$, we identify an accessible system control parameter p (which in this instance is $H_{\rm dc}$). When the system state point begins to depart the vicinity of the unstable fixed point along its unstable manifold, a small time dependent change is made to this parameter such that the next iterate will fall onto the stable manifold of the unstable fixed point. According to OGY this control parameter shift is

$$\delta p_n = C(\boldsymbol{\xi}_n - \boldsymbol{\xi}_f)\cdot \mathbf{f}_u, \quad (1)$$

where $\mathbf{f}_u$ is the unstable contravariant eigenvector, $\boldsymbol{\xi}_n=(\xi_n,\xi_{n-d})$, and C is defined as

$$C=\frac{\lambda_u}{\lambda_u-1}\frac{1}{\mathbf{g}\cdot\mathbf{f}_u}. \quad (2)$$

λ_u is the unstable eigenvalue of the system and $\mathbf{g}\equiv\partial\boldsymbol{\xi}_f(p)/\partial p$ is the change in the position of the fixed point given a change in the control parameter. All of the parameters that comprise C are determined experimentally in the usual fashion [9].

Once control is established, the tracking is begun by advancing (or drifting) our chosen tracking parameter $H_{\rm dc}$ a small increment (0.0034 Oe). Because of our applied "drift" in $H_{\rm dc}$ the value of the fixed point about which we are currently controlling ($\boldsymbol{\xi}_f$) will not be the correct value for the new tracking parameter value. To check the closeness of the fixed point about which we are currently controlling to that of the "true" fixed point, we examine the mean of the last 10 values of the control parameter. If the value of the current control fixed point is correct, then the mean of the fluctuations of the control parameter will be zero, within experimental error. If it is not zero, one needs to correct the fixed point about which we are currently controlling in such a way as to minimize this mean. Schwartz and Triandaf relate the fixed point position to the fluctuations in the control parameter:

$$\langle \delta p_n\rangle = C\langle \boldsymbol{\xi}_n\cdot\mathbf{f}_u\rangle, \quad (3)$$

where $\langle \delta p_n\rangle$ is the mean value of the fluctuations in the control parameter. For our experiment we have previously shown these fluctuations to have a Gaussian distribution [10]. Thus the definition of C given by Eq. (2) implies that the correction to the fixed point is

$$\delta\xi_f=\frac{\langle \delta p_n\rangle}{C(f_u^x+f_u^y)}. \quad (4)$$

If the system were linear, this would suffice to place us exactly at the correct value of the fixed point. In practice this equation is iterated until the control converges to the correct value.

The effects of noise in the system and of uncertainties in determining the quantities that go into the estimate of C may cause Eq. (4) to overestimate the location of the "true" fixed point with a subsequent loss of control. To guard against this, we replace it with the following equation:

$$\delta\xi_f=\frac{\langle \delta p_n\rangle}{CV}, \quad (5)$$

where V is a number that is chosen experimentally to improve the estimate. To be conservative we set $V=4$. This modification has the added benefit of guaranteeing that convergence to the correct value for the fixed point is taken in smaller steps than those given by Eq. (4), preventing the loss of control during tracking. With this value we never lost control.

The difficulty with estimating the correct value of the fixed point arises because we are using a linear estimator. Schwartz and Triandaf employ a nonlinear predictor-corrector method to get their estimate. However, this particular method, at least when implemented in our experiment, is sensitive to system noise and to uncertainties in the constituent quantities of the constant C and thus renders the estimate of the fixed point unusable in our experiment. A more robust nonlinear estimator would serve to improve this procedure.

During stabilization, the mean of the perturbation distribution and its standard deviation are calculated for 10 iterates. We then check the mean against the standard deviation. If the mean is more than the standard deviation (thus ensuring that the mean is statistically different from zero), the position of the fixed point is corrected according to Eq. (5); otherwise the process is deemed complete.

Once the fixed point has been determined for the second value of the tracking parameter, subsequent values are initially estimated as

$$\xi_f^{n+1}=\xi_f^n+(\xi_f^n-\xi_f^{n-1}) \quad (6)$$

and then refined according to the procedure outlined above.

An enhancement to the above procedure is suggested by Eq. (1). Since the quantities that are represented by the constant C, namely λ_u, $\mathbf{f}_u$, and $\mathbf{g}$, are changing as we change the tracking parameter, they should be redetermined at each step along the way. Indeed Petrov and co-workers similarly redetermine λ_u in their experiment at each new value of the tracking parameter. However, this type of optimization requires that control be lapsed during the process. In order to maintain continuous control, we resort to the following optimization procedure. After the tracking process has converged to the "true" fixed point for a new tracking parameter, we vary C slightly from its current value and examine the resultant change in the standard deviation of the distribution in the controlled points around the "true" fixed point. C is varied in fixed step sizes of 1% until the standard deviation is minimized (a more sophisticated technique could employ a variable step size if optimal convergence is required). The value of C that minimizes the standard deviation is then taken to be the new value of C.

Figure 1(a) shows the results of this tracking procedure as applied to an unstable period-1 fixed point. Starting from within the chaotic regime (since ergodicity guarantees that the neighborhood of the desired unstable fixed point will eventually be visited), this point is controlled and tracked through the chaos (through both decreasing and increasing tracking parameter $H_{\rm dc}$), into the period-doubling region and finally into the period-1 region where the fixed point is

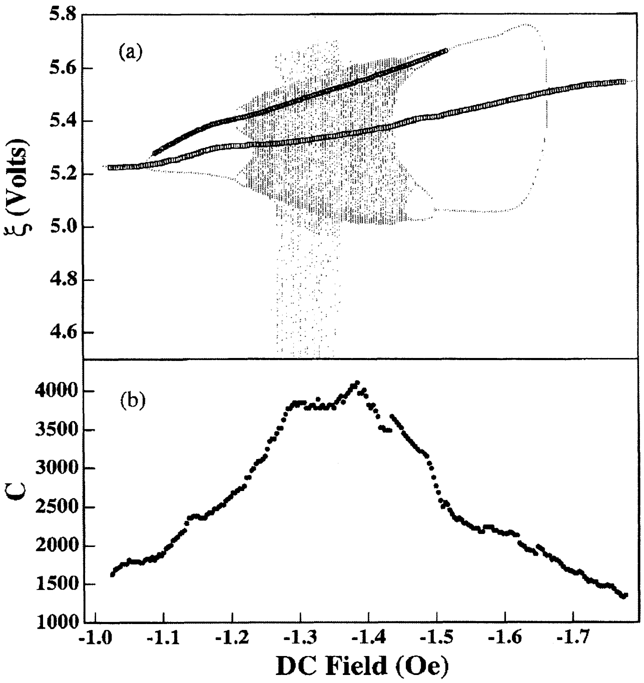

FIG. 1. (a) The bifurcation diagram (gray) for the magnetoelastic ribbon at $f = 1.18$ Hz and $H_{ac} = 1.05$ Oe. The lower dark line denotes the tracked period-1 fixed point starting from the dc field where the fixed point is unstable in the chaotic region, through the bifurcation regions on either side to where the fixed point becomes stable. The upper dark line similarly marks the tracked period-2 orbit. (Only the upper branch is shown.) Note the agreement between the tracked fixed points and the natural period-1 and period-2 fixed points (in the regions where they are stable). (b) The variation of C as a result of optimizing the control during the tracking shown in (a). Note the correspondence of the attractor blowup with the plateau between -1.25 and -1.35 Oe.

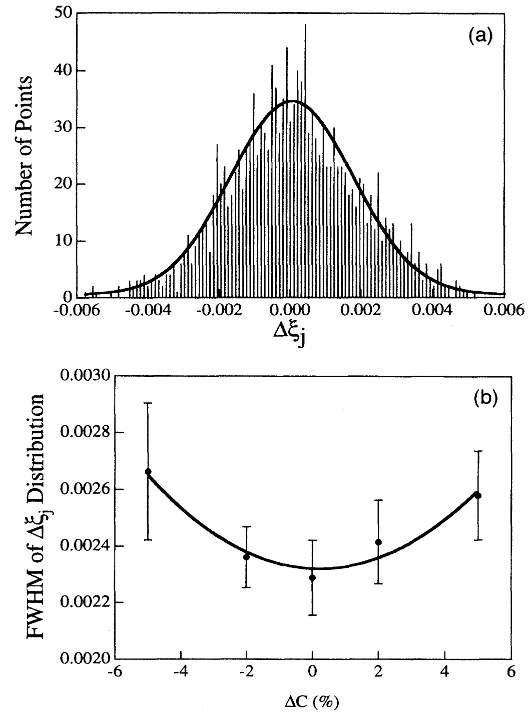

FIG. 2. (a) The distribution of the ribbon position about the control fixed point for the iterations after the correct fixed point has been determined in a typical run. The fit of this data to a Gaussian distribution is shown by the heavy line. The width of this distribution is a function of the control feedback parameter C, as shown in (b). (b) The width of the Gaussian for several different changes of the value of C by $\Delta C\%$. The data have a well defined minimum that we use to correct the value of C to improve the control. The solid line shows a quadratic fit to the data.

stable. Note that the excellent agreement between the tracked period-1 point and the uncontrolled period-1 point in the latter region assures us that the tracking and optimization are working correctly. This tracking procedure is accomplished with no loss of control over the whole range of "drift" or tracking parameter region.

Figure 1(b) shows the variation in C obtained from our optimization process for the data in Fig. 1(a). It is interesting that the plateau between -1.25 and -1.35 Oe corresponds to the attractor blowup seen in Fig. 1(a) (for which we have no explanation).

Also shown in Fig. 1(a) is the result of tracking a period-2 orbit (upper leg shown). Note that the control, and hence the tracking, were only applied about one of the two saddles of this unstable orbit. Again note that there is strong agreement between the tracked fixed point and the natural period-2 fixed point in the period-2 region.

Figure 2(a) shows the distribution of the deviations from the control fixed point at each iteration (after the correct fixed point has been determined) during the optimization of the value of C. This value of C is 4% larger than the original value of C. The solid line is a fit of this data to a Gaussian distribution. Figure 2(b) plots the width of the fitted Gaussian for five different values of C. The value of C at the minimum is taken as the new C. (In the case shown, no adjustment to C was necessary. More typically changes in C of $\pm 5\%$ or less were required for optimization of control.)

It is anticipated that tracking unstable periodic motions in nonstationary systems will have application to mechanical, optical, and electrical systems. Additionally the demonstrated robustness and computational simplicity of our adaptive tracking could also provide a key improvement in the control of chaos in such typically nonstationary biological dynamics as the beating of hearts [11] and seizures in brains [12]. Indeed tracking may prove indispensable for the maintenance of long term control of chaos in engineering or biological systems where loss of control could prove catastrophic.

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